Harmonic RF Cavity for Bunch Lengthening & Shortening Ch1,Ho3

In the presence of both a fundamental and a harmonic cavity the net voltage is given by,

$$eV(\phi) = eV_0\left[\sin(\phi+\phi_s) + k\sin(N\phi + N\phi_N)\right],$$

where k and ϕ_N are free parameters to be determined by the effect desired from the harmonic cavity.

The equation describing the longitudinal motion is given by

$$\ddot{\phi} + \frac{\Omega_{s0}^2}{eV_0\cos\phi_s}\left[eV(\phi) - U_0\right] = 0, \quad \Omega_{s0}^2 \equiv \frac{\alpha eV_0\cos\phi_s\omega_{RF}}{ET_0}.$$

A first integral of this equation is given by,

$$\frac{\dot{\phi}^2}{2} + \frac{\Omega_{s0}^2}{\cos\phi_s}U(\phi) = \frac{\dot{\phi}_{max}^2}{2},$$

where $U_0 = eV_0\sin\phi_s$ and the effective potential is given by,

$$U(\phi) = \frac{1}{eV_0}\int_0^{\phi}\left[eV(\phi') - U_0\right]d\phi' = -[\cos(\phi+\phi_s) - \cos\phi_s +$$

$$\frac{k}{N}\cos(N\phi + N\phi_N) - \frac{k}{N}\cos N\phi_N + \phi\sin\phi_s].$$

The free parameters k & ϕ_N can be chosen to modify the effective potential to lengthen the bunch by making the leading term in the potential quartic instead of quadratic or the quadratic term can be enhanced to shorten the bunch length. A summary of the results is given in the following table.